

# Trigonometry - Advanced problems

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Problem	Solution 1	Solution 2	Solution 3
1. 1963 AHMSE Problem 34	✓		
2. 1973 AHMSE Problem 17	✓		
3. 1988 AHMSE Problem 13	✓		
4. 1989 AHMSE Problem 14	✓		
5. 2003 AMC Problem 21	✓		
6. 2004 AMC Problem 21	✓		
7. 2006 iTest Problem 9			
8. 2006 iTest Problem 17	✓	✓	
9. 2007 AMC Problem 17			
10. 2007 iTest Problem 17	✓	✓	✓
11. 2008 iTest Problem 46	✓		
12. 2017 AMC Problem 7	✓		
13. 2020 AMC Problem 9			
14. 1993 U South Carolina Problem 10			
15. 1984 AIME Problem 13			
16. 1991 AIME Problem 9			

## Problems

### 1963 AHSME Problem 34

1. In triangle  $\triangle DEF$ , side  $d = \sqrt{3}$ , side  $e = \sqrt{3}$ , and side  $f > 3$ . Let  $t$  be the largest number such that the magnitude of the angle opposite side  $f$  exceeds  $t$ . Find  $t$ .

### 1973 AHSME Problem 17

2. If  $\alpha$  is an acute angle and  $\sin(\frac{\alpha}{2}) = \sqrt{\frac{x-1}{2x}}$ , then find  $\tan(\alpha)$ .

### 1988 AHSME Problem 13

3. If  $\sin(j) = 3 \cos(j)$ , then what is  $\sin(j) \cdot \cos(j)$ ?

**1989 AHSME Problem 14**

4. What is  $\cot(10) + \tan(5)$ ?

**2003 AMC Problem 21**

5. An object moves 8 cm in a straight line from  $A$  to  $B$ , turns at an angle  $\beta$ , measured in radians and chosen from the interval  $(0, \pi)$ , and moves 5 cm in a straight line to  $C$ . What is the probability that  $AC < 7$ ?

**2004 AMC Problem 21**

6. If  $\sum_{n=0}^{\infty} \cos^{2n} \delta = 4$ , what is the value of  $\cos 2\delta$ ?

**2006 iTest Problem 9**

7. If  $\sin(x) = -\frac{5}{13}$  and  $x$  is in the third quadrant, what is the absolute value of  $\cos(\frac{x}{2})$ ?

**2006 iTest Problem 17**

8. Let  $\sin(2x) = \frac{1}{7}$ . Find the numerical value of  $\sin(x)\sin(x)\sin(x)\sin(x) + \cos(x)\cos(x)\cos(x)\cos(x)$

**2007 AMC Problem 17**

9. Suppose that  $\sin(a) + \sin(b) = \sqrt{\frac{5}{3}}$  and  $\cos(a) + \cos(b) = 1$ . What is  $\cos(a - b)$ ?

**2007 iTest Problem 17**

10. If  $x$  and  $y$  are acute angles such that  $x + y = \frac{\pi}{4}$  and  $\tan(y) = \frac{1}{6}$ , find the value of  $\tan(x)$ .

**2008 iTest Problem 46**

11. Let  $S$  be the sum of all  $x$  in the interval  $[0, 2\pi)$  that satisfy  $\tan^2(x) - 2\tan(x)\sin(x) = 0$ . Compute  $[10S]$ .

**2017 AMC Problem 7**

12. The functions  $\sin(x)$  and  $\cos(x)$  are periodic with least period  $2\pi$ . What is the least period of the function  $\cos(\sin(x))$ ?

**2020 AMC Problem 9**

13. How many solutions are there to the equation  $\tan(3x) = \cos(x/2)$  on  $[0, 2\pi]$ ?

**1993 U South Carolina Problem 10**

14. What is  $\arcsin(\frac{1}{3}) + \arccos(\frac{1}{3}) + \arctan(\frac{1}{3}) + \operatorname{arccot}(\frac{1}{3})$ ?

**1984 AIME Problem 13**

15. What is  $10 \cot[\cot^{-1}(3) + \cot^{-1}(7) + \cot^{-1}(13) + \cot^{-1}(21)]$  ?

### 1991 AIME Problem 9

16. Suppose that  $\sec(x) + \tan(x) = \frac{22}{7}$  and that  $\csc(x) + \cot(x) = \frac{m}{n}$ , where  $\frac{m}{n}$  is in lowest terms. What is  $m + n$ ?

- a) What are some trig identities which would be helpful for this problem?
- b) How could we set up systems of equations for this problem? If we have two equations with 2 different pieces of information, how could we set them up?
- c) How can we write  $\sin(x)$  as a multiplication of other trig functions?
- c) How do we know if a number is divisible by 3? 4? 5? 6? 7? 8? 9?
- d) Find the prime factorization of 841 (*Hint: List of prime numbers, 2, 3, 5, ... 29*)
- e) Find the prime factorization of 435