

Algebra 2

Sample plan - Horace McDonald

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Book: *Algebra 2* by James Schultz, et. al © 2004 , ISBN 0-03-070044-2

Topic	Problem 1	Problem 2	Problem 3
1 Numbers and relations			
1.1 Number sets	✓	✓	✓
1.2 Properties of addition and multiplication	✓	✓	
1.3 Properties of Exponents			
2 Linear and polynomial functions			
2.1 Linear functions	✓	✓	
2.2 Properties of functions			
2.3 Function transformations			
3 Systems of equations and inequalities			
3.1 Classifying systems	✓	✓	
3.2 Solving by elimination	✓	✓	
3.3 Systems of inequalities	✓	✓	
4 Matrices			
5 Complex numbers			
	✓		

Topic	Problem 1	Problem 2	Problem 3
6 Rational and radical functions			
7 Exponential and logarithmic functions			
7.1 Exponential functions	✓		
7.2 Logarithmic functions			
8 Conics			
8.1 Intro	✓	✓	
8.2 Parabolas			
8.3 Ellipses	✓	✓	✓
8.4 Hyperbolas			
9 Sequences and series			
	✓		
10 Probability and statistics			
	✓	✓	
11 Vectors			
	✓		

Problems

1.1 Numbers sets

1. List or state the definition of:

- a) The set of whole numbers
- b) The set of natural numbers
- c) The set of integers
- d) The set of rational numbers
- e) The set of irrational numbers
- f) The set of real numbers

1.2 Properties of addition and multiplication

1. There are different properties of addition and multiplication. State each property for addition and multiplication:

- a) Closure
- b) Commutativity

c) Associativity

d) Identity

e) Inverse

2. Using the numbers d , e , and f provide an example of the distributive property.

3. Let b be a real number. Then if n is a natural number, state the definition of

$$b^n$$

4. If d is a real number, what is d^0 ?

1.3 Properties of exponents

1. Let b and c be non-zero real numbers. Let p and q be integers. Provide an example of each:

Product of powers	$(b)^p(b)^q = b^{p+q}$
Quotient of powers	
Power of a power	
Power of a product	$(bc)^p = b^p c^p$
Power of a quotient	

2. For all real positive numbers b , what is another way to write:

a) $b^{\frac{1}{m}}$ if m is a non-zero integer?

b) If p is an integer and m is a non-zero integer, what are different ways to write $b^{\frac{p}{m}}$?

2 Linear and polynomial functions

2.1 Linear functions

1. What is the slope of a line?

2. What is the slope-intercept form of a linear equation?

3. What is the standard form of a linear equation?

4. What is the point-slope form of an equation?

5. What is the slope when two lines are parallel?

6. What is the slope when two lines are perpendicular?

7. Write an equation with y and x which is a direct-variation equation. Plot two different points, and show $\frac{y}{x}$ for the points.

2.2 Properties of functions

1. What is the domain of a function? Provide an example with a function.
2. What is the range of a function? Provide two examples where the ranges of two different functions are not the same.
3. What is the vertical line test? What is it used for?
4. What is a relation in math? Are all functions a relations? Are all relations a function? Give an example of a function which is a relation and a relation which is not a function.
5. Provide an example of function notation. Which is the independent variable? Which is the dependent variable?

[*Challenge*] What is an example of function notation which is not rectangular?

6. For all functions g and f , fill in the table with the operation name and equality

Sum		$f(x) + g(x)$
Difference		
	$(f \cdot g)(x)$	
		$\frac{f(x)}{g(x)}$

7. What is the composition of functions g and h ? What is $h(g(x))$? [*Hint, you will use a small circle.*]
8. If $g(x) = x^2 + 1$ and $h(x) = \frac{1}{x}$, what is $g(h(x))$? $h(g(x))$?
9. What is the inverse of a relation? What is the inverse of a function?
10. What is the domain of the inverse of a relation? What is the range of the inverse of a relation? [*Hint, think about x and y 's*]
11. What is the horizontal line test used for?
12. If a function passes the horizontal line test, what is the domain of the inverse of the function? What is the range of the inverse of a function? [*Hint, think about x and y 's*]
13. If p and q are functions, and $p(q(x)) = q(p(x))$ then p and q are said to be
14. If $h(x) = 4x + 5$ and $v(x) = \frac{1}{4}x - \frac{5}{4}$, show $h(v(x)) = v(h(x))$. What does this mean?
15. Come up with an example of a simple piece-wise function.
16. Graph the function $f(x) = \lfloor x \rfloor$.
17. Graph the function $w(x) = \lceil x \rceil$.
18. We can write the absolute value function as a piece-wise function. What is the piecewise function?
19. Say that $f(x)$ is a relation, such that $f(x) = |x|$. Graph $f(x)$ and $f^{-1}(x)$.

2.3 Transformations

1. If $y = g(x)$, what is the general form of a vertical translation for $g(x)$? If (x, y) is a point on the original graph, what is this point on the new graph?
2. You are given the function $g(x)$ and want to shift all points down 3 units. What is g_{new} ? If (x, y) is a point on the original graph, what is this point on the new graph?
3. For a function $f(x)$, what is the general form of a horizontal translation for $f(x)$? If (x, y) is a point on the original graph, what is this point on the new graph?
4. You are given the function $f(x)$ and want to shift all points to the right 2 units. What is f_{new} ? If (x, y) is a point on the original graph, what is this point on the new graph?

Vertical and horizontal scaling

5. For a function $h(x)$, what is the general form of a vertical stretch of factor k , where $k > 1$? If (x, y) is a point on the original graph, what is this point on the new graph?
6. You are given the function $h(x)$ and want to stretch all points by a factor of 3. What is h_{new} ? If (x, y) is a point on the original graph, what is this point on the new graph?
7. For a function $j(x)$, what is the general form of a vertical compression of factor p , where $0 < p < 1$? If (x, y) is a point on the original graph, what is this point on the new graph?
8. You are given the function $j(x)$ and want to compress all points by a factor of 7. What is j_{new} ? If (x, y) is a point on the original graph, what is this point on the new graph?
9. For a function $m(x)$, what is the general form of a horizontal squish of factor t , where $t > 1$? If (x, y) is a point on the original graph, what is this point on the new graph?
10. You are given the function $m(x)$ and want to squish all points horizontally by a factor of 10. What is m_{new} ? If (x, y) is a point on the original graph, what is this point on the new graph?
11. For a function $i(x)$, what is the general form of a horizontal compression (spreading out) of factor w , where $0 < w < 1$? If (x, y) is a point on the original graph, what is this point on the new graph?
12. You are given the function $i(x)$ and want to horizontally spread all points by a factor of 5. What is i_{new} ? If (x, y) is a point on the original graph, what is this point on the new graph?

Reflections

1. If you have the function $a(x)$, how do you reflect it across the x axis?
2. Reflect $r(x) = (x - 2)^2 + 4$ across the x axis.
3. If you have the function $b(x)$, how do you reflect it across the y axis?
4. Reflect $r(x) = (x - 2)^2 + 4$ across the y axis.

3 Systems of equations and inequalities

3.1 Classifying systems

1. What are the three different ways we can classify a system?
2. Graph and classify the system:

a)

$$\begin{cases} x + y = 3 \\ 3x + 2y = 4 \end{cases}$$

b)

$$\begin{cases} 2x + 4y = 3 \\ 6x + 12y = 9 \end{cases}$$

c)

$$\begin{cases} 3x - 5y = -1 \\ 9x + 7 = 15y \end{cases}$$

3.2 Solving by elimination

1. Solve the system by elimination:

$$\begin{cases} x + 7y = 7 \\ -3x + 4y = -11 \end{cases}$$

Examples

1. Solve $5 + 2x = 5x - 11$. Check your solution.

3.3 Systems of inequalities

2. Solve the equation and draw solution on a number line.

$$4x - 15 \leq 9$$

3. Solve the equation and draw solution on a number line.

$$4x + 23 > 8$$

4. Solve the compound inequality and graph the solution on a number line.

$$5y \leq 15$$

and

$$y + 7 > 2$$

5. Solve the compound inequality and graph the solution on a number line.

$$3w \geq 21$$

or

$$2w + 5 < 6$$

6. Solve the absolute value equation, and plot the solution on a number line.

$$|3x + 2| = 5$$

7. Solve the absolute value equation, and plot the solution on a number line.

$$|7x + 8| = 2x + 6$$

8. Solve the absolute value inequality, and plot the solution on a number line.

$$|4 - 3x| < 5$$

4 Matrices

5 Complex numbers

6 Rational and radical functions

7 Exponential and logarithmic functions

8 Conics

8.1 Intro

1. List the four basic conics.
2. List the three degenerate conics.
3. What is the general second-degree equation for a conic?
4. What is a locus?

8.2 Parabolas

1. What is the quadratic form of the equation for a parabola along the vertical axis? State axis of symmetry.
2. What is the quadratic form of the equation for a parabola along the horizontal axis? State axis of symmetry.
3. What is the standard form of the equation for a parabola along the horizontal axis? State vertex, focus, and directrix.
4. What is the standard form of the equation for a parabola along the horizontal axis? State vertex, focus, and directrix.
5. What is the vertex form of the equation for a parabola along the vertical axis? State vertex, focus, and directrix.
6. What is the vertex form of the equation for a parabola along the horizontal axis? State vertex, focus, and directrix.
7. What is the equation for a parabola with focus (4,3) and directrix $y = -3$?
8. What is the standard equation for a parabola with vertex at the origin and focus (4,0)?

8.3 Ellipses

1. What is the definition of an ellipse? State in your own words.
2. Draw out an ellipse, labeling the 2 foci, a point on the ellipse, the distance from the foci to the point, the vertices, co-vertices, major and minor axes, semi-major and semi-minor axes, with a, b, c, h, k

3. What is the relation between a , b , and c ?
4. What is the equation for an ellipse whose major axis is horizontal?
5. What is the equation for an ellipse whose major axis is vertical?
6. Find the standard form of the equation of an ellipse which has foci at $(6, 1)$, $(0, 1)$ and major axis of length 8.
7. What is the eccentricity of an ellipse? What is the eccentricity of the ellipse in problem 6?

8.4 Hyperbolas

1. What is the equation for a hyperbola? How are a , b , and c related in this case?
2. Find the equation for a hyperbola with foci at $(4, 3)$, $(0, 3)$ and vertices $(3, 3)$, $(1, 3)$.

9 Sequences and series

10 Probability and statistics

11 Vectors